Chapter 2  Formal systems in architectural design

Formal systems in architectural design aim at the systematic description, interpretation, and evaluation of existing works of architecture as well the systematic creation of new works of architecture. A basic algorithmic structure for the foundation of formal systems is reviewed (Stiny and Gips 1978) and various examples of such systems are presented. The chapter concludes with an informal presentation of applications in formal analysis and design based on group theory.

2.1. Introduction

‘What makes me tick is an aesthetic sense of order, of essential simplicity behind apparent complexity. As an artist, it is possible to create exuberant and unique objects from a small and limited set of elements and rules; as a scientist, it is a challenge to discover a simple explanation for complex behavior, a general causal structure for a series of related but unique events. In this view, science and art are both aesthetic activities: only the direction of the approach differs.’ March (1972).

The desire to speculate architectural design as a form of a logical construct has a long history (for informative accounts of various attempts see for example, (Stiny and March 1981), (Kalay 2004). Particularly interesting are the efforts in the 1960’s to formalize architecture in terms of some mathematical framework when design methods in architecture were associated with operational research, economics and decision theory (see for example (Archer 1970), (Martin 1967), (Simon 1994). Integral in such a world-making is the construct of reasoning as the process of extending a set of known facts, beliefs or observations by applying to them rules that combine the known facts in a manner that produces new facts and rules. Lionel March (1976) suggested Peirce’s three modes of inference as the three possible plausible reasoning in science and in design (March 1976), (Shin 1994). Whereas the major goal of scientific endeavor is to establish general laws or theory, the prime objective of designing is to realize a particular case or design. Both require deduction for analytical purposes. Yet science must employ inductive reasoning in order to generalize and design must use productive inference so as to particularize. These two modes of reasoning can be distinguished by the role the hypothesis plays. The outcome of productive
reasoning is the case that is called the design or composition; the outcome of deductive reasoning is a decomposition which comprises the characteristics of the design that emerge from analysis of the whole composition; and the outcome of inductive reasoning is a supposition, a working rule of some generality – a model. Such a speculative design cannot be determined logically, because the mode of reasoning involved is essentially abductive. It can only be inferred conditionally upon our state of knowledge and available evidence. Deductive methods can then be used to predict measures of expected performance applicable to the particular design proposal. Concerning the question of value, a design in itself has no value. It assumes relative value through comparison with other designs. As such, evaluation assumes that suppositions about worth, preference, desirability or utility can be inferred. These suppositions form the substance of the productive phase of designing. Thus, the models required to produce design alternatives are value-laden. Therefore, ‘value theory is the essential foundation of any rational theory of design’ (March 1976). As Peirce writes, abduction, or production, is the only logical operation that introduces any new ideas. Induction does nothing but determine a value, and deduction merely evolves the necessary consequences of a pure hypothesis. Figure 2-1 shows March’s adaptation of Peirce’s ideas.

Figure 2-1: March’s modes of inference and the PDI-Model (March 1976)

For the team of architect-scientists in Cambridge, the bridges between architecture and other disciplines were firmly established. Upon the concept of modeling designed and described by the
Cambridge philosopher Hesse (1966) in his book ‘Models and Analogues in Science’, models, quantitative techniques, structuralism were all up driving the discourse. By 1968, the research group thrived on producing the one model after the other, the one list of equations after the other, and on firmly establishing the usage of mathematics in architecture and planning. In that sense, mathematics itself defined in the most general sense as a science dealing with definition and manipulation of symbolic models became an indispensable ally to architecture, a science and art dealing equally with definition and manipulation of pictorial and symbolic models.

Still, the focus here is not to give a definite account of this history of formal systems in architecture. Instead one system will be used extensively to provide the scaffolding for the presentation of various general characteristics pertaining to their structure and usage in architecture. This system, Stiny and Gips’ ‘algorithmic aesthetics’, is used here because of its generous structure that a) deals equally with various art forms; b) addresses both analysis and design; c) is built upon both the constructive and evocative modes of understanding; and d) makes extensive usage of the idea of algorithm and computation and therefore formalizes all above issues. All definitions here follow closely Stiny and Gips’ conception of aesthetics as the philosophy of both criticism and design in the arts, a definition in itself extending Beardsley’s conception of aesthetics as metacriticism, otherwise known as philosophy of criticism (Stiny and Gips 1978). The work here reviews the original model by Stiny and Gips (1987), and two more models associated with it, the ‘design machine’, a model for design by Stiny and Gips (1981) and the ‘vitruvian machine’ (Economou and Rieher, 2008), a recent adaptation of the model based on a mapping of the design machine upon the Vitruvian triad (Morgan, 1914) in the earliest surviving account of architecture discourse. The review here concludes with several precedents and applications from architecture discourse cast within these systems and especially case studies drawn from applications of group theory in formal analysis and composition in architecture design.

2.2. Formal systems

‘Representation that is verbal is classical. By contrast, visual representation is non-classical because of its lack of primitives...Both kinds of representations are interrelated and blurred at the boundaries... The non-classical ones made with things
Formal systems in architecture are concerned with questions about how existing works of architecture can be described, interpreted, and evaluated and with questions about how new works of architecture can be created. A formal system dealing with description, interpretation and evaluation of an existing work of architecture is called an ‘analysis system’. A formal system dealing with the creation of a new work of architecture is called a ‘synthesis system’ or design system. In this sense formal systems are dealing with questions of criticism and design in architecture, with the foundations of criticism and design in architecture and in essence with the philosophy of criticism and design in architecture.

The medium of all such formalisms is computation. A nice account of computation has been given recently by Knight and Stiny (2001) wherein two aspects of computation, ‘representation’ and ‘process’ are considered as generators of and as species of computation at large; representation has to do with the way objects in a computation are described and process has to do with the rules that are used to carry it out. A basic division of representation in verbal and visual kinds (‘classical’ and ‘non-classical’ vocabularies) and a corresponding division of process in terms of explanation and results (classical processes if the results are understandable in terms of the rules and non-classical if the opposite) produces basically four categories of computation. These categories are then combined under a basic schema of representation/process to produce the following four categories of computation: a) classical/classical computation; b) classical/non-classical computation; c) non-classical/classical computation; and d) non-classical/non-classical computation. The four categories are shown pictorially in Figure 2-2.

![Figure 2-2: Four categories of computation with respect to representation and process](image-url)
2.3. The structure of formal systems

The original schema for algorithmic aesthetics proposed by Stiny (1978) postulates a structure for criticism and design of works of art based on informational process models of thought. The basis of the structure postulated for Stiny’s criticism algorithms and design algorithms is modeled after Kenneth Craig’s model of thought (1943). Craig model consists of three essential properties: a) translation of external processes into symbols; b) arrival at other symbols by processes of reasoning; and c) retranslation of these symbols into external processes. Figure 2-3 shows the basic structure of Craig’s model of thought.

![Figure 2-3: A diagrammatic representation of Craig’s model of thought](image)

2.3.1. Aesthetics machine

The basic novelty of Stiny’s model with respect to Craig’s model – essentially an input-output construct – is that it requires a third component to augment the basic structure of computation. This new component is called an ‘aesthetic system’ and is situated in-between the typical input-output schema of Craig’s model of thought (1978). A second powerful novelty of the model, already mentioned so far, is the postulation of an identical structure of a formal system for criticism and design in arts; in both cases the formal system consists of four components, a receptor (input), an effector (output), an aesthetic system distinct from both input and output, and an analysis or synthesis algorithm (theory) that uses in different ways the three other parts. Analysis systems and design systems share the same structure. The basic components of these systems are three: a receptor, an aesthetic system and an effector. The task of an analysis system is to produce a response to an architecture object as a work of art; the task of a design system is to produce an architecture object as a work of art with respect to some initial conditions. The core of
both is the design of the aesthetic system. A diagrammatic representation of the Stiny’s model for criticism and design in the arts is shown in Figure 2-4.

![Diagram of Stiny and Gip’s formal model for criticism and design in the arts]

Figure 2-4: A diagrammatic representation of Stiny and Gip’s formal model for criticism and design in the arts

The receptor contains a list of descriptions of events, objects or processes of the outside world. Objects and events have an infinity of properties that may be of interest but the ones that are encoded in the receptor are only those that are matching the given requirements and bias of the machine. This list may contain a finite sequence of symbols encoding texts, drawings, images, sounds, numbers, and so forth. The receptor consists of two parts, a transducer and a linked algorithm to encode the output of the transducer into a description consisting of symbols. The transducer can be a television or infrared camera, a microphone, textual survey responses, two-dimensional or three-dimensional scanner, a satellite recorder, and so forth. Less fancy but infinitely more complicated receptors are own personal sensory machinery – eyes, ears, hands, and so forth. The complexity of the structure of the receptor depends on the complexity of the design of the transducer and the linked algorithm. The output of the receptor can be very straightforward as in a bitmap array of color values of a scene or very complex as in a textual description of a scene. Furthermore, the relationship of the external event or process and the description of the receptor cannot fixed; different receptors may describe the same process in different ways and different processes may be described in a similar way by one receptor.

The effector contains a list of instructions to produce a response to the receptor. The effector consists of two parts, an algorithm to convert a description of design – set of drawings, datasets, texts and so forth– to instructions to produce the result of the computation and a transducer to instantiate the design. The transducer can be a two-dimensional or three-dimensional printer, a two-dimensional or three-dimensional numerically controlled milling machine, a robot to
assemble parts, a speaker, and so forth. Less fancy but infinitely more complicated effectors are our own personal motor machinery – hands, legs, muscles, voice and so forth. The complexity of the structure of the effector depends on the complexity of the design of the transducer and the linked algorithm. The output of the effector can be very straightforward as in a printed bitmap of a scene or very complex as in a painterly description of a scene.

The aesthetic module of the formal system is the heart of the whole construct. This system contains a finite sequence of symbols encoding texts, drawings, images, sounds, numbers, and so on, and more specifically it includes descriptions of all possible designs of a certain kind. Each language (set of designs) may be defined in terms of some fixed point of interest, say the Palladio designs, and each may be contain diverse descriptions such as three-dimensional models, drawings, or diagrams. Languages may be ordered in any desired degree of complexity defining elaborate structures cutting across spatial and temporal boundaries. Each language may be defined strictly by enumerating the designs in the set or by identifying rules for their generation. The key idea is that aesthetic systems exist independently of other considerations and that their use and value in a computation depends upon the fitness between them and a criticism or design inquiry.

Finally the theory component of the design machine is the link connecting the other three components of the machine; it determines the fit between a design and a design context defined by a receptor and effector. Essentially the theory supplies the principles that enable a design machine to choose the most suitable design for a design context.

### 2.3.2. Design machine

The design machine is an adaptation of the aesthetic machine specifically for the constructive mode of understanding in spatial design and its goal is the specification of an algorithmic structure for design (Stiny and March 1981). The rules in this system depend on three things: a) the rules given to compose designs, i.e., to construct their descriptions; b) the rules given to describe designs in other terms pertaining to their intended meaning and purpose, or the way they are connected to a complex of associations and ideas; and c) the rules given to assess the quality of designs in terms of the way we compose them or the way we describe their meaning and purpose, and so on. The original aesthetic module is substituted here from the language of designs
and the analysis or design algorithm used respectively for criticism or design is specifically here substituted by the theory module. The diagram of the design machine is shown in two different versions in Figure 2-5; the first emphasizes the main logical connections among components. The second stresses the relationship among components giving an emphasis on the receptor and the effector as the design context of a given design inquiry.

![Figure 2-5: Two diagrammatic representations of the Stiny and March’s design machine](image)

2.3.3. Vitruvian machine

The vitruvian machine (Economou and Riether, 2008) is a formal model of architecture composition and analysis that partially extends the existing model for criticism and design mentioned so far in two significant ways: a) the model correlates both the aesthetics machine and the design machine with architecture discourse and particularly -and polemically too- with the earliest surviving treatise on architecture and the three Vitruvian categories of architectural form; and b) the model exemplifies its partition to map with existing architectural discourses and to suggest a generous theoretical framework for analysis and design in architecture discourse.

More specifically, this formal system is mapped upon the earliest model of architecture discourse surviving in the writings of Vitruvius (Morgan, 1914) and his account of the three principles of architecture, the categories of ‘venustas’, ‘firmitas’ and ‘utilitas’ - typically translated as beauty, firmness and commodity. These categories of description, interpretation and evaluation of form directly allude to the Aristotelian foundations of this work and the corresponding interpretative framework of architecture in terms of geometric, material and functional characteristics respectively. The mapping between the two models is isomorphic. The receptor is mapped to
utilitas (commodity) (U) and to function broadly conceived to include technical specifications, performance specifications, engineering specifications and so on. The effector is mapped to firmitas (firmness) (F) and to materiality broadly conceived to include all technology specifications and production specifications. The aesthetics module of the aesthetics machine or the language module of the design machine is mapped to venustas (beauty) (V) and to geometry broadly conceived to include all pictorial and spatial descriptions of form. Figure 6 shows the diagrammatic representation of the isomorphism between the design machine and the Vitruvian categories and the resultant diagram for architecture design termed here the Vitruvian machine.

![Figure 2-6: A diagrammatic representation of the Vitruvian machine.](image)

This diagram for design suggests a complete structure for alternative definitions of design processes. The structure of the design processes and their corresponding formal exercises is derived by a set theoretical analysis of the framework of the pedagogy. The possible combinatorial subsets of the Vitruvian triad are $2^3 = 8$ including the empty set. Including the empty set that suggests a null input and response, the possible theoretical constructs for criticism and design are eight. These constructs are nicely mapped to existing discourses of architecture that inform one another. Furthermore, these eight modules can be structured in three phases that correspond to the three subsets of distinct ordinal numbers for the Vitruvian set and suggest a rising complexity in the discourse of architecture; excluding the study with null input-output $\{O\}$, three modules isolate one element of architecture discourse $\{V\}$, $\{U\}$, $\{F\}$, three modules are comprised by two elements $\{V,U\}$, $\{V,F\}$, $\{U,F\}$, and the last one is the complete triplet $\{V,U,F\}$. A brief presentation of all eight modules in formal criticism and composition in architectural design follows below.

The null set does not specify any action for criticism or design. This null set with its rather zen qualities can be mapped to discourses for criticism or design where there are no instructions nonesover, no deliverables and where everything goes. Figure 2-7 illustrates the null Vitruvian machine.
The second module foregrounds the domain of a geometric vocabulary as the context for formal criticism and design studies. The module foregrounds the history and logic of geometry in the description and construction of space. The formal vocabularies in this module are abstract geometrical terms including points, lines, planes, triangles, squares, circles, conic curves, Bezier curves, NURBS, and so forth, as well as evocative spatial descriptions such as porosity, permeability, balance, symmetry, proportion, order, disorder and so on. Figure 2-8 provides a diagrammatic representation of the Vitruvian machine foregrounding geometry or shape.

The third module foregrounds the domain of a functional vocabulary as the context for formal criticism and design studies. The module foregrounds the history and role of function in the description and construction of space. The formal vocabularies are functional definitions, constraints, conditions, and relations. Figure 2-9 provides a diagrammatic representation of the Vitruvian machine foregrounding function.
The fourth module foregrounds the domain of material vocabulary as the context for formal criticism and design studies. The materiality of form - hard, soft, elastic, rough, smooth, opaque, transparent, translucent, and so forth, supports, enables foregrounds or even contradicts the criticism and the design description. This module explores the affordability of a variety of different materials to render diverse possibilities of criticism and design and suggests a systematic exploration of materiality and fabrication methods. Figure 2-10 provides a diagrammatic representation of the Vitruvian machine foregrounding materiality.

![Figure 2-10: A partial Vitruvian machine foregrounding materiality](image)

The fifth module pairs function and materiality and proposes a loop of formal criticisms and design explorations informed by functional and material considerations. Some functional arrangements are specifically enabled by specific materials and some materials afford different functional organizations to emerge. The loop between the two domains suggests two different trajectories of reasoning, one starting from function and testing against materiality (UF) and the other way around (FU). The partition is designed to explore the interrelation of programmatic organizations and material properties without taking into account geometrical or shape considerations. Figure 2-11 provides a diagrammatic representation of the Vitruvian machine foregrounding function and materiality.

![Figure 2-11: A partial Vitruvian machine foregrounding function and materiality](image)
The sixth module pairs function and geometry and proposes a loop of formal criticism and design explorations informed by functional and geometrical considerations. Some spatial arrangements are apt to allow specific functions and some functions often emerge in specific spatial organizations. The loop between the two domains suggests two different trajectories of reasoning, one starting from function and testing against geometry (UV) and the other way around (VU). The partition is designed to explore the interrelation of programmatic organizations and formal languages without taking into account material or construction considerations. Figure 2-12 provides a diagrammatic representation of the Vitruvian machine foregrounding function and geometry.

![Figure 2-12: A partial Vitruvian machine foregrounding function and geometry.](image)

The seventh module pairs geometry and materiality and proposes a loop of formal criticism and design explorations informed by geometrical and material considerations. Some systems of geometry are informed by specific models of construction and some construction techniques are developed to meet geometric demands. The loop between the two domains here suggests two different trajectories of reasoning, one starting from geometry and testing against materiality (VF) and the other way around (FV). The partition is designed to explore the interrelation of design specifications and fabrication methods, including aspects of prototype structures, form and formwork, scalability and so forth. Figure 2-13 provides a diagrammatic representation of the Vitruvian machine foregrounding geometry and materiality.

![Figure 2-13: A partial Vitruvian machine foregrounding geometry and materiality.](image)
The eighth exercise closes the design inquiry suggested by the Vitruvian machine and fully engages all three aspects of the model. The loop between the three domains here suggests \(3! = 6\) different trajectories of reasoning: VUF, VFU, FVU, UVF, UFV, and UFV. Every theoretical trajectory selected has to be understood, reflected and critiqued upon the ways it informs and it is informed by the other theoretical trajectories of the design process. And still, all trajectories should be present in the end suggesting a totality and complexity that resent unpacking and command alternative interpretative discourses. The partition is designed to allow for a full immersion in architecture discourse with complete sets of programmatic requirements, performance specifications, technical specifications, engineering specifications, production specifications and so forth. Figure 14 shows the complete Vitruvian machine.

![Figure 2-14: A complete Vitruvian machine.](image)

**2.4. Analysis and design systems**

One of the central claims of the original aesthetics machine is its applicability in both criticism and design. The formal systems that can be constructed upon this function are a) the analysis systems; and b) the design systems. A formal system dealing with analysis requires as an input component a description of an architecture object and, as output component, a statement about some formal properties such as type, arrangement, symmetry, rhythm, proportion as so on. Analogously, a formal system dealing with design requires, as input component, some rules or data with schemata that particularize these data and as output component a description of an architecture object. In both systems, interpretations are defined independently of actual architecture objects; descriptions of architecture objects are manipulated and co-related to give an interpretation of these objects in terms of associations or constructive rules.
2.4.1. Analysis systems

An analysis system has, as an input component, a description $\lambda$ of the object and, as an output component, a statement about some formal properties $\beta$ such as symmetry, proportion, balance, or rhythm. This formal system is generally described by an algorithm of the form $\langle \lambda, \beta \rangle$ wherein $\lambda$ is the description of an object and $\beta$ is the list of evocations generated by the description $\lambda$; this algorithm specifies how an object with description $\lambda$ is understood by listing the properties $\beta$. In general, in a system of this type the input component might be a list of descriptions of a building, including pictorial data such as plans, sections, elevations, sketches or photos or symbolic data such as texts, tables or any other form of symbolic analyses, and the output component might be a statement about structure, arrangement, rhythm, or symmetry. A series of examples that can be modeled in this fashion follow below.

An example of this system in design is March and Steadman's (1971) approach in the analysis of houses by Frank Lloyd Wright's. Widely diverse designs are topologically equivalent and share the same underlying structure; topological transformations of the geometry of three houses which is generated by repeated applications of different geometric units. Units are composed of an equilateral triangle, a square and a circle that produce three individual designs which share the same underlying structure. The plans and the underlying graph that shows the correspondence of the various subspaces in the houses is shown in Figure 2-15.

Figure 2-15: Three topologically equivalencies by F. L. Wright –a) graph; b) Sundt; c) Life; d) Jester

It is quite interesting that this specific formal analysis provided the blueprint for a host of other similar types of formal analyses that all sought to exhibit the common transformational structure that links various types of design. The very same formal analysis as above is repeated by Laseau (1992) in other case studies including the Life house, the Hanna house, and the variations of the Jacobs’ house all by of F.L.Wright and is given in Figure 2-16.
Another example in formal analysis is March’s analysis of the ratios in R.M Schindler’s How House in Los Angeles, California. The partition of the whole plan or various parts of the house, such as the piano nobile or the maid’s wing, can be seen as a straightforward recursive application of a family of rectangle of specific ratios associated with music discourse (March 1993). The main construct used in his analysis is the fact that any rectangle characterized by these musical ratios can be divided into rectangles with corresponding musical ratios. A sequence of partitioning of the maid’s wing suggests a musical development of the plan and is illustrated in Figure 2-17.
A very different but exciting example is Birkoff’s (1933) attempt to commensurate the aesthetic value of form – in fact any form. The ensuing formula (see below), arresting in its simplicity, imposes a basic analogical relationship between the characteristics of order (O) and complexity (C) and introduces various other parameters to address idiosyncrasies of various modalities of form. The general form of the aesthetic measure (M) is given in (1).

\[ M = \frac{O}{C} \]  

(1)

For example, for the case of the rectangle the various parameters that enter the computation are vertical symmetry (V), equilibrium (E), rotational symmetry (R), relation to horizontal and vertical network (HV) and unsatisfactory form (F). The aesthetic measure (M) of the polygons for specific relation among these parameters, is then given in (2).

\[ M = \frac{O}{C} = \frac{V + E + R + HV - F}{C} \]  

(2)

The actual computation of these parameters provides an ordering scheme for the arithmetical ratios of similarly positioned rectangles and provides a specific framework of interpretation of rectangular forms suitable for specific types of composition. It is worth noting that Birkhoff (1933) drew upon the experiments of the psychologist Fechner (1860), (1876) who ascertained that the most satisfactory series of rectangular shapes, including the square, is the sequence within the range of one-to-one and one-to-two presented in Figure 2-18.

Figure 2-18: Five rectangles with ordered ratios \( r \) by Birkoff’s aesthetic measure.
2.4.2. Design systems

A design system has, as an input component information $\alpha$ needed to construct an object and as an output component a description $\lambda$ of an object. This spatial system is generally described by an algorithm of the form $<\alpha, \lambda>$ whereas $\alpha$ is the information needed to construct the object and $\lambda$ is the description of an object; the algorithm specifies how an object is understood by listing the information $\alpha$ that generate the description $\lambda$ of an object. The information given by the input may be considered as a list of instructions to be followed or as a list of data to be acted upon. In the first case the rules provide all the necessary information to construct the object, and in the second case the data have to be acted upon by a schema encoded within the system.

The input component is considered as a list of instructions when it entails an explicit provision of primitive elements and rules for the combination or organization of the elements; in this case the rules are applied to the elements and result in the description of the object. An example of this formal system in spatial design is Froebel's 'kindergarten method' for the construction of simple designs using a series of geometrical 'gifts' and a system of 'categories' of geometrical forms (Stiny 1980). In this pedagogical system, a series of simple geometrical shapes are given to the children along with some rules of combinations to create designs defined in a system of categories.

In general, in a formal system of this type the input component might be any list of primitive spatial elements and a list of rules that specifies how the parts are combined; the output component would be the description of a building in plan, elevation, section or any three-dimensional perspective view that conforms to the imposed rules. In a formal system of this type dealing with the form of music, the input component might be a simple motive and the rules for generating the piece from that motive, and the output component would be the score of a piece or any other description of a piece.

The input component is considered as data when it gives explicitly the primitive elements that are developed or arranged according to a schema for a large class of descriptions of a certain type. An example of a system of this type in spatial design is Durer's schema for the description of the human face. In this system each individual face is a 'parametric transformation' of a standard schema; the data particularize the proportions of a dimensionless grid and produce descriptions of different faces which all fit the schema.
Examples of design systems for which the input component is a list of primitive spatial elements and a list of rules that specifies how the parts are combined have been nicely captured by the shape grammar formalism (Stiny and Gips 1978), Stiny (1976, 1985, (1990), 1991, 1992, Knight, 1994b) and especially by the ‘kindergarten grammars’ (Stiny 1980). The latter is a type of spatial algorithms that formalizes the pedagogical character of Froebel's kindergarten method and extends the notion of construction of languages of designs (sets of shapes) from scratch (Stiny, 1981, Knight, 1992, 1994a). As already stated earlier, this formal system uses a series of geometrical ‘gifts’ and a system of ‘categories’ of geometrical forms and it is a formalization of a pedagogical system for the training of the children invented by Frederick Froebel, wherein a series of simple geometrical shapes are given to the children along with some rules of combinations to create designs defined in a system of categories. Figure 2-19: A simple shape grammar consisting of a) one initial shape and b) one shape rule. shows a kindergarten grammar consisting of simple shape, one labeled rule and a design in the pre-specified language.

Figure 2-19: A simple shape grammar consisting of a) one initial shape and b) one shape rule.

An example of data that parameterize a specific schema is Sullivan’s approach to design. This approach can be embedded in the web of his ideas on functionalism. For him, design includes functions that satisfy cultural and higher spiritual necessities of humankind, and not just the utilitarian needs for which ‘form follows function’ has mistakenly been cited (Sullivan 1922). Figure 20 shows Sullivan’s generation of a series of motifs based on the structure of the square.

Figure 2-20: Sullivan’s square motifs
2.5. **Constructive and evocative systems**

A second central claim of the original aesthetics machine is that analysis and design in aesthetics use two different models of understanding that each suggests a profoundly different world-making. More specifically, it is suggested that aesthetic systems can be characterized and computed in terms of their interpretations. Typically objects can be understood or interpreted in terms of a) how they can be constructed, and b) what associations, ideas and emotions they evoke. The former systems are referred to as ‘constructive systems’ and the latter as ‘evocative systems’. Any other system of interpretation can be based on any combination of these two basic types.

The fundamental formal distinction between the two systems is that the description of the object $\lambda$ is the output component of the computation in the constructive systems, whereas in the evocative systems, it is the input component. A typical example of a constructive system is the understanding of a number sequence in terms of the rules used to generate it. An interpretation of an object has the form $<\alpha, \lambda>$, where $\alpha$ is the list of rules to produce the description $\lambda$, or a schema for a large class of descriptions of a certain type. A typical example of an evocative system is the understanding of a number sequence. It is like a telephone number and a corresponding list of associations involving the person with this telephone number. An interpretation of an object has the form $<\alpha, \lambda>$, where $\alpha$ is the list of rules to produce the description $\lambda$, or a schema for a large class of descriptions of a certain type.

This distinction between constructive and evocative understanding can be used in both analysis and design systems to produce basically four different structures for criticism and design: a) a constructive – analysis system; b) an evocative – analysis system; c) a constructive – design system; and d) an evocative – design system. A series of applications based on these formal systems are presented below.
2.5.1. *Constructive systems*

*'Using elementary set theoretical notions, together with simple proportional structures, I found that I could compose in my head quite elaborate works, develop them, and construct related series under various transformations and permutations.*'

March (1972)

Constructive systems are mathematical models based on Pierce’s modes of inference: deterministic models, probabilistic models, and creative models. The first ones are deductive, the second ones are inductive, and the third ones are abductive. The rationale of adopting these models during the search of design solutions is shown below.

A series of constructive spatial systems is presented here using tools from set theory, group theory, graph theory, Boolean algebra, permutations and shape grammars. All systems entail some mathematical model involving a class of undefined elements and relations between these. All these models reproduce suitable chosen features of the physical situation if it is possible to establish rules of correspondence between specific environmental elements and corresponding mathematical elements and relations in the models, that is, if it is possible to have what is technically known as an isomorphism between the two domains.

A system relating the organization of space with graph theory to produce generic house plans has been given by Steadman (1971). The model suggests that it is possible to determine the most probable distribution of spaces independently of any particular arrangements. Figure 2-21 shows the isomorphism between graph theory and room layout.

![Figure 2-21: Use of isomorphism and graph theory in house planning](image)
A different application of graph theory, using models from the theory of electrical networks, produces a graph that represents the adjacencies of the relative positions of the rooms in the plan, and their exact dimensions and shapes (Bullock 1971). Figure 2-22 shows an illustration from the original model: by translating the course options into a switching circuit, a loose fit approach can be tailored to limit the space and time to accommodate the course requirements.

Figure 2-22: Switching circuits – a) circulation patterns; b) activities modeled
Applications of probability models are numerous. A probability assignment is a numerical encoding of a state of knowledge. The rationale of this approach is straightforward: a) Write down as much as you know about the system of interest, encoding this knowledge in a set of equations; b) infer, mathematically, the most likely state of the system on the basis of the given information while maximizing your impartiality; c) Compare your inferences with your observations. Differences between observed and expected represent either what you do not know, or what you know but have failed to make explicit. Generalization is inductive and it consists in perceiving possible general laws in the circumstances of special cases.

A nice spatial system using this construct is a zoological study of cells by Weiss (1955). Figure 2-23 shows a diagram from the application of the model representing a system of order defined by the linear distribution of a three-by-three magic square numbers mapped into a twenty-five-square grid. The cells symbolize the range within which a dot inside is free to roam – a range that has nine degrees of freedom – responsibility to move within a constraint orbit.

Another example of such models is Conway’s game of life where interaction between cells is based on adjacency requirements (Wolfram 1984). An illustration of such interaction typically modeled by the mathematics of cellular automata is given in Figure 2-24. A cellular automaton is an algorithm for generating a set of cells given a prior set of cells (Wolfram 1984). An isomorphism between one-dimensional and two-dimensional cellular automata and settlement patterns in Africa has been recorded by (Eglash 2005).
A different system using tools from Boolean algebra in architectural design is March’s (1976) Boolean description of built form encoded into his minimal representation schemes. In this type of representation a plan is mapped upon a dimensionless grid whose combinations of empty and filled cells signify the arrangement of spaces or the arrangement of walls and so on. For example this representation allowed March (1976) to argue that the Mies’ Brick house, the Gropius’ Dessau building and the Schindler’s King’s Road house appeared to share a similar structure of a butterfly motif with extending wings, and more specifically that the Mies’ house and the Schindler house shared an identical Boolean representation – March’s Lectures Notes (Economou 1992). Figure 2-25 shows the original plans and their minimal representations.

A similar example of a formal description utilizing tools and methods of set theory in architectural design is March’s (1971) generation of the ground plan of the ‘maison minimum’. Figure 2-26 shows the derivation of the overall plan; walls and doorways are shown in plan and axonometric projection.
A second application of set theory in formal composition is March’s (1966) serial art where the operations of union and intersection on one basic generator define the whole sequence of spatial motifs. The whole series of arrangement of the structural row and its variations based on a unit and its inverse, their unions and intersection as well as their transformational relations through reflection and rotation are shown in Figure 2-27. The composition may have been sparked by Albers’ square frames.

![Figure 2-27: Two compositions – a) Albers’ square frames; b-c) March’s set theoretical composition based on rigid motions and union-intersection operations](image)

Another formal constructive system is the Galois representation and the Hasse diagrams that both use aspects of formal ordering to classify objects and properties. In this representation design objects are represented as elements of a set and relations and/or operations on these elements and the result is derived from an algebraic model of that design. A rather involved but rich example of usage of partial order lattices and group theory is Park’s (2000) representation of the Free Public Library by R. M. Schindler, Jersey City, 1920 with all its subshapes of the first-floor plan partially ordered with the full symmetry of a square. Error! Reference source not found. illustrates how the lattice of subsymmetries of the square of the Library is constructed.

![Figure 2-28: Park’s lattice representation of the subsymmetries of Schindler’s Free Public library - a) plan and elevation; b) subsymmetries](image)
An extension of this graph theoretical representation that captures order in design has been given by Economou for the construction of original classes of design using group theoretical tools and three-dimensional spatial elements (Economou 2001; 2007; 2008). Figure 2-29 shows the generation of a three-dimensional study that consists of the sum of all possible ten symmetry subgroups found within the structure of the dihedral group of order four; details about symmetry groups and their constructs are given later in this chapter as well as in chapter three.

Figure 2-29: A generative description of a three-dimensional $D_4$ house.

2.5.2. Evocative systems

‘... A mask operated as a palimpsest mysteriously guiding the location of the walls. The space is thus a dislocation induced by the forms of the masks. The effect is to create within the rational space of the grid a violent juxtaposition of perplexing spaces. The villa Stein at Garches was then considered as the prototype of modern architecture. Rather than a simple fetish, the mask here served as subversion for the order of reason through its spatial implications.’ Tschumi (1976) in (Midgard no1)

Evocative systems are integral components of the aesthetic machine and are used both for analysis and synthesis systems. Various case studies are briefly presented below that belong in either domains and point either to a criticism of existing architecture or to the making of new.

Russian constructivism provides an incredible array of case studies that all foreground a host of diverse aspects of formal characteristics in their construction or associations (Khan-Magomedov 1983). Malevich’s ‘suprematism’ and El Lissitzky’s ‘prouns’ amplify the cubist measures of portraying space, and suggest separate avenues for manipulating and heightening the spatial effects of abstract form, as does Rodchenko’s spatial compositions with line and plane
(Senkevitch 1983). Like the Cubists, Malevich (1917) seeks to return to the pristine elements of form, shapes such as squares, rectangles, circles, and triangles. Unlike the Cubists, he reduces the number of elements, increases their size, and eliminates all traces of representation, creating nonobjective compositions.

Emerging as a logical consequence of Malevich’s work, the Prouns of El Lissitzky’s bridge painting to architecture by conveying explicit spatial depth through three-dimensional renderings of entities imbued with architectonic clarity. Not only do elements of Lissitzky’s composition move back and forth, as do those of Malevich’s paintings, but they convey a dynamic concept of kinetic form and space. Thus, the notion of simultaneous projection and penetration in irrational space and the concept of expressing form and space through the rhythmic grouping of elements in imaginary space is analogous to the three-dimensional space to be formulated as a conceptual matrix for their spatial form (Lissitzky 1968).

Drawing upon predecessors, Rodchenko creates a system of pictorial construction in which the process of manipulating elements, rather than the shapes themselves, becomes the focus of perceptual activity (Senkevitch 1983). Rodchenko’s compositions for the first time create the definite impression of being constructed rather than composed and so, give birth to Constructivist aesthetic. ‘Each line, in itself, neither carries any particular esthetic impact nor implies any pictorial space. And yet, spatial effects of considerable power and elaboration are achieved through the interaction of lines to create transparent planes possessing visual density, scale, and tangible spatial definition’ (Senkevitch 1983). A series of suprematist constructions are shown in Figure 2-30.

![Figure 2-30: Spatial form in suprematism. a) Suprematist drawing (1917); b) Proun 1A (1919); c) Artist’s compass (1915)](image)

Another interesting evocative approach is the one described by Giedion (1967) in his Space, Time and Architecture whereas he considers formation of space as one out of two archetypal house-forms: mass and wall. From these two categories all systems receive their basic ordering. Mass
systems must be external representations of an internal volumetric order. They convey the conception of a generic solid that has been eroded or cut away as in the Villa Moissi (Flemming 1978). However, the mass can also be thought of as having been juxtaposed with a series of volumetric planes as in Le Corbusier’s Villa Stein (Eisenman 1963). To achieve any systemic organization the vocabulary (volume, mass, surface and movement) must be ordered and therefore clarified by a grammar. Since volume is that property of generic form put forward as being fundamental to any architectural expression, some form of volumetric ordering will occur in any system, without necessarily providing the basis for that system. Moreover, in situations where a volumetric order is dominant, this order can either be continuous or static: both applicable to either a centroidal or a linear situation. There is a third type of volumetric order which is conceived of as a series of volumetric planes.

First, in a continuous system, the movement or circulation is interconnected with the volumetric ordering. The continuous system is associated with early modern avant-garde architecture. Thus we find examples of continuous volumetric ordering in the De Stijl house projects, and the early Cubist and Purist exercises. In De Stijl, the volumetric system is combined with a surface of planar system while in Purist work the volumetric ordering is related to the system movement. Second, in a static system, each volume is expressed or articulated as an individual entity. A sense of total organism is achieved by means of a sequential progression: volumes are linked together as beads on a string. Third, a series of vertical volumetric planes has three recognizable subtypes of volumetric systems. These subtypes depend upon a surface ordering for their definition; the essential characteristic is the volumetric ordering that relates to a surface or juxtaposition of surfaces for its reference. The first subtype is a series of vertical volumetric planes tensioned from a vertical reference, illustrated by Le Corbusier’s Garches. The composition is ordered by a series of volumetric planes defined initially by the front façade. The second subtype, a series of horizontal volumetric planes, is defined by a sequence of horizontal surfaces, tensioned usually from an articulated floor or roof plane: the prototype of this being the Maison Domino. The third subtype of mass-surface system is the volumetric plaid which derives its order from two adjacent surfaces: one horizontal and one vertical plane in juxtaposition. The villa Shodhan by LC combines the principles of the Maison Domino with those of Garches to produce the resulting plaid.

An example that nicely illustrates this spatial distinction is Flemming’s (1990) interpretation of Loos’ architectural vocabulary as a playful composition of these fundamental properties of volume, mass, and surface. A cubic volume is carved out of its mass by hollowing out interior
space. Subtractive operations are used to carve into the surfaces of the remaining walls, creating profiles, and finally windows and doors. Figure 2-31 shows Flemming’s interpretation of Loos’ Villa Moissi at Lido as a case study of mass and surface architecture.

Another archetypal house-form is the study of walls as independent elements considered as pure form and analyzed by Padovan (2002). This leads to neo-plasticism in which rectangular panels are placed in the three major directions to form an abstract composition that follows some ordering principles. The basic element manipulated is a rectangular panel placed vertically or horizontally, frontally or across. Windows and other openings are given by the gaps between panels. To close the gaps, a second, transparent panel is needed. Van Doesburg’s Counter-Construction of Eesteren’s Private House (1923) is a case study that foregrounds planarity of spatial elements as the principal organization of form. Van Doesburg explores the implications of the Eesteren house design in a series of analytical counter-constructions in which the solid volumes are isolated in disjoint classes: vertical planes – panels, horizontal planes – slabs, and cubic volumes – collage elements. This technique to isolate the elements of composition is an unequivocal example of syntactic layering and an example of surface ordering. Rules must take into account both ways of operating and show first how new panels are added, then the insertion of transparent panels once solid panels have been placed and finally that of linear and volumetric elements. An extension of these rules can set the generation of truly three-dimensional compositions with the use of functionally equivalent interchangeable parts. Figure 2-32 shows Padovan’s reproduction of Doesburg’s counter-construction from Eesteren project as an example of surface architecture.

Figure 2-31: Loos’ Villa Moissi at Lido: mass and surface architecture
A different form of abstraction occurs when mass and surface are combined to define a plane or layer that provides a datum for organizing a composition. Rowe (1972) asserts that parallel layers organize buildings such as Le Corbusier’s Villa Stein at Garches. They establish links with the principles of Purist Aesthetic. Whereas Mies assembles freestanding elements in empty space, Le Corbusier carves its voids out of the solid square frame inwards (Padovan 2002). A variation of the vocabulary of this language contains another basic sub-type: columns and beams within a structural frame and infill elements that generate enclosures and spatial divisions. The frame establishes the whole and divides it into clearly related parts with some logic for placement of spatial divisions. As it is exposed at the inside, it affirms overall ordering function, and continuity with the outside. Figure 2-33 shows both Eisenman’s (1963) and Kulic’s (1999) interpretation of Corbusier’s Villa Stein at Garches as an example of layered architecture.

The evocative distinctions need not be restrained in orthogonal geometrical systems. An architecture of forms that is geometrically non-Euclidean, presents a great opportunity of sculptures that become buildings despite the complex negotiations among figure, structure, and program. The introduction of such amorphous figures into architecture provokes new connections with conventional representation, technique, and context. There has been an extra-Euclidean counter-tradition that has managed to survive through history. It shows up in ancient knowledge.
at Sheba-Sirwah (temple of the moon, 1000BC) (Clapp 2001), and Zimbabwe (Imba Huru temple, 1400AD) (Walton 1953) and Yoruba Compound (Fassassi 1978), as well as in modern twentieth century architecture such as in Philharmonie (Sharoun 1995), the German Embassy in Brazilia (Syring 2004), the Ronchamp Chapel (Clark 2005) and the Ulm Building (Meier 1991) and so forth. A series of buildings that foreground and at the same time resist their interpretation in terms of spatial and planar layering is shown in Figure 2-34.

![Figure 2-34: Three Sculpture-buildings: a) Le Corbusier; b) c) Meier](image)

Typical cases of evocative systems include all the systems that associate forms between different symbolic systems and especially those between architecture and painting. In modern painting, there are two methods to work out physical form: abstraction and collage. Leger’s (1972) and Kandinsky’s (1979) work shows how forms of abstraction de-familiarize everyday objects through the manipulation of geometry. Figure 2-35 shows how the human form used as a plastic element helps to simplify the geometric order and how the inner relation between a complex of straight lines and a curve is achieved in the work of the ‘Mechanical element’ by Leger (1972) and the ‘Black triangle’ by Kandinsky (1979).

![Figure 2-35: Mechanical Element and Black triangle](image)
This process of formal de-familiarization brings the object into the sphere of new perception. De Stijl too follows this path in the creation of new object world. Objects are described by pure formal relationships derived from formal universals based on geometric abstraction. Figure 2-36 shows two different sets of images, one on abstractions of African heads (Din 2003) and the second on a abstraction of an organic form to geometric form with the case study of the cow by Van Doesburg (Zimmer 2003).

Figure 2-36: Abstraction as an evocative system - African’s Head and van Doesburg’s Cow

Collage is either a device for assembling a picture from diverse fragments of discarded materials, or the result of a process of experimentation, notation, and operations. Any collage emphasizes the unity between formal logic and pictorial composition. As a device, literal collage is involves the juxtaposition of physical material, whereas phenomenal collage involves the ambiguity and reciprocity of figure/field (Hildner 1997). Figure 2-37 shows a) a free-hand pencil drawing that deconstructs the Gris’ painted collage maps the white space-defining fragments; b) a cut-figure shadow image of the ‘Guitar Player in Profile’ that exemplifies the Cezannesque organizing principle of interlocking cut figures; c) the form of the interlocking with a Woman Listener locked to the Guitar Man along the fault-line for shallow-space | deep-space.

Figure 2-37: Collage as an evocative system. Still Life with Guitar, Juan Gris 1917

Aalto (1940) calls this mode of operation: ‘the Purist overemphatic rhythm in which all impulses run parallel’. This quality of imposed order endows a transparent organization of the capacity of
multiple readings of the interconnections between the parts of a whole system. When the figures are endowed with transparency, they interpenetrate without an optical destruction of each other. Transparency implies more than an optical characteristics, it implies a broader spatial order. Transparency means a simultaneous perception of different spatial locations and it may be an inherent quality of substance or it may be an inherent quality of organization. Figure 2-38 shows two examples of layering process.

![Figure 2-38: Layering as an evocative system. a) Jeanneret’s Still-life; b) Hoesli’s decomposition](image)

2.6. **Languages and configurations**

One key idea of the constructive systems is that the information given by the input may be considered as a list of instructions to be followed or as a list of data to be acted upon. In the first case the rules provide all the necessary information to construct the object; in the second case, the data have to be acted upon by a schema encoded within the system. The first case may be referred as a ‘language’ and the second as a ‘configuration’ (Stiny and March, 1981; 1984).

This last part of this exposition of formal methods will deal primarily with the configurations aspect of the constructive systems. A central point in the ‘Geometry of Environment’ (March and Steadman 1971) is that design is a mode of computation that explicitly exercises both imagination and reason. In this sense a formal theory of spatial design is directly linked to Alberti’s (1486) worldview and provides a syntactic procedure for creation of designs based on speculative knowledge. The configurations themselves, that is the schemata that particularize the data, can be modeled after syntactic and semantic domains; the syntactic domains are typically given in terms of arithmetic and geometry respectively; the semantics under functional ones. These three categories of dealing with configurations can be nicely mapped to the Froebel categories of form:
a) forms of knowledge to which we relate quantities of modern proportion and spatial relations; b) forms of beauty to which we attribute qualities of spatial transformations and symmetries; and c) forms of life to which we order and configure the spatial representations of actual objects with functional semantic meaning attached to them (March 1992). This mapping can be furthered to relate well-known existing categories of formal inquiry in architecture design such as proportion, symmetry and compartition with arithmetical, geometrical and semantic elements and relationships and this is in fact the mapping given in the area of studies known as ‘architectonics’ (March 1998, Economou 1998). A brief exposition of the first two domains is given here, proportion and symmetry, to prepare the ground for the formal exposition of the model introduced in this thesis.

2.6.1. Proportion

Ratio and proportion have been significant concepts in architectural design since the first surviving treatise in architecture design by Vitruvius (Morgan 1914) and still were dominant fields of inquiry throughout the twentieth and twenty-first century. Ratio is a relation between two numbers and a proportion is a relation between two ratios. The last number that can establish a proportion is three. For three numbers \(x\), \(y\) and \(z\) and \(x < y < z\) there are three possible outcomes of comparisons, one unique case of equality \(x:y = y:z\) and two cases of inequality \(x:y < y:z\) and \(x:y > y:z\). For each case of inequality, there can be an infinite number of subcases with respect to the actual value of the number involved in the comparison. Among these relationships some are more interesting than others. For example for three numbers \(x\), \(y\) and \(z\) and \(x < y < z\) if \((1/z) - (1/y) = (1/y) - (1/x)\) the inequality can be written as an equality, namely, \((z - y)/z = (y - x)/x\). The problem has been nicely solved in antiquity by Greek mathematicians in a series of successive attempts initially proposing two more such equalities by Archytas, the arithmetic and the harmonic ones, later three more, possibly by Eudoxus, and finally two additional distinct sets of four by Nichomachus and Pappus respectively, with three overlapping cases between them, bringing the total number of equalities to ten. These ten relationships of ratios plus the initial one of equality, the geometric mean, brought the number of comparisons to eleven and they are all treated informingly under the heading of proportionality theory or theory of means (Heath, 1921). Figure 2-39 shows three out of eleven ways of comparing two ratios involving three magnitudes, namely the arithmetic, geometric and harmonic ratios.
All root ratios can be nicely depicted by a shape grammar of adding successively squares starting with an initial shape of a square (March, 1998). If the derivation of the grammar is shown in a tree, several sets of ratios with spatial properties are depicted including in the extremes the unit ratios such as 2:1, 3:1, 4:1, 5:1 to the left, and the Fibonacci ratios 2:1, 3:2, 5:3, 8:5 to the right as well as all the. Nichomachean ratios such as the multiplex, superparticular, superpartiens, multiplex superparticular, and multiplex superpartiens (Heath 1921). The initial derivation of the grammar is shown in Figure 2-40.

Ratios and proportion provide a very rich vocabulary for systematic studies in formal composition and March and several others have offered a considerable body of work foregrounding arithmetical relationships in form analysis and synthesis. Among these case studies the Schindler’s system for the How House has been prominently featured as a case study in twentieth century architecture that exemplifies principles of classical composition. Figure 2-41 shows few orthographic projections of the R.M Schindler How House.
2.6.2. Symmetry

Symmetry as we understand it today is radically different from the Greek word ‘symmetria’ from which it derives. A nuance of the old meaning of ‘symmetria’ still survives in the new context and is nicely illustrated in the above quotation by Hermann Weyl from his classic book on symmetry. To the Greeks ‘symmetria’ (συμμετρία < συν+μετρον: with measure) meant commensurability and it was suggested to be a canon of beauty in nature and in art. Two magnitudes are said to be commensurable if there exists a third magnitude that divides them both without remainder. As applied to works of art, symmetry meant commensurability of the parts of a work to one another and to the whole; in other words, a work of art was considered symmetrical if all the parts were exact multiples of a visible part of this work, a module. A rather blurred account of this notion of symmetry was given by the Roman architect Vitruvius in his treatise ‘De Architectura’.

"Symmetry is a proper agreement between the members of the work itself, and relation between the different parts and the whole general scheme, in accordance with a certain part selected as standard...In the case of temples, symmetry may be calculated from the thickness of a column, from a triglyph, or even from a module; in the ballista, from the hole or from what the Greeks call the περιτρητος; in a ship, from the space between the tholepins (διαπηγμα); and in other things, from various members"...

..."The design of a temple depends on symmetry, the principles of which must be carefully observed by the architect. They are due to proportion, in Greek αναλογια. Proportion is the correspondence among the measures of the members of an entire
work, and of the whole to a certain part selected as standard. From this result the principles of symmetry”. (Vitruvius, [c.50 BC] Morgan, 1914, 14 and 72)

Both definitions quoted above illustrate Vitruvius’ attempt to give an appropriate definition of symmetry and its relation to proportion; the first passage refers to the fundamental principles of architecture and the second passage refers on principles involved in the design of temples. Symmetry was asserted to be the key to perfection; the canons of antiquity were attempts to capture this idealized beauty by imposing an order and a rationale in their construction. This intellectual conception of beauty as guaranteed by symmetry and expressed in the Pythagorean and Platonic doctrines defined a line of thought which still pervades the formulation of various compositional techniques in architecture and music.

Another conception of symmetry came in the foreground in Renaissance with the doctrine of the Golden Section, or as it was then generally known, the ‘Divine Proportion’ or ‘Sectio Aurea’ (Paccioli, 1509). In this context, symmetry results from the systematic application of a single proportion; this proportion is essentially the proportion between two relations that have one term in common and one of the three terms is the sum of the other two terms. This proportion arises when a line is divided in extreme and mean ratio, that is, in such a way so that the ratio of the whole line to the greater line is equal to the ratio of the greater line to the smaller line. This symmetry, based on the extreme and mean ratio, was asserted to be the universal key to perfection in nature and in art; it sacrificed exact commensurability but it imposed a single ratio throughout. According to this definition, an object was assumed to be symmetrical when all its parts were related to one another and to the whole not by means of commensurable ratios as in ‘symmetria’, but by means of one single incommensurable ratio, namely the $\phi$. Inherent in this conception of symmetry were the beliefs that the same principles of perfection apply in nature and in art with the additional doctrine that the ideal symmetry was conditioned mathematically by the Divine Proportion. This kind of symmetry was revived in the middle of the nineteenth century by A. Zeising (1954) and since then it comes and goes in the foreground of the scientific and artistic milieu by means of writings and works of researchers and artists. However, these works and books, even if they have been influential they are not convincing as to the universal and ubiquitous role of the extreme and mean ratio.

The modern conception of symmetry developed around the notion of repetition and is a strictly geometric and precise concept. The origins of the modern conception of symmetry and the shift
from the concept of ‘symmetria’ are to be found in the writings of the Florentine architect and theoretician Alberti and especially, in his treatise on the art of building, ‘De re Aedificatoria’:

"Look at Nature's own works...right should match left exactly. We must therefore take great care to ensure that even the minutest elements are so arranged in their level, alignment, number, shape, and appearance, that right matches left, top matches bottom, adjacent matches adjacent, and equal matches equal... I have long been an admirer of the ancients in which they displayed outstanding skill: with statues, especially for the pediments of their temples, they took care to ensure that those on the one side differed not a whit, either in their lineaments or in their materials, from those opposite". (Alberti, [1486] Rykwert et al, 1988, 310).

Bilateral symmetry, that is, the identical disposition of a theme or a motif about both sides of an imaginary axis, is just one of several types of symmetrical configurations that are composed by identical parts. In this case, the bilateral symmetry of the design is induced by a reflection about a mirror line or a mirror plane passing through the middle point of the design. Other transformations that induce symmetrical designs are rotations, inversions, translations, glide reflections, screw rotations and others. Bilateral symmetry, that is, the identical disposition of a theme or a motif about both sides of an imaginary axis, is just one of several types of symmetrical configurations that are composed by identical parts. The incorporation of these isometries in the study of symmetry occurred gradually. The concept of symmetry has undergone substantial changes over the time. Originally, and very much in the same line of thought with Alberti's principles, the symmetries of shapes were related exclusively with mirror reflections in planes. Simple rotation axes were added later to the symmetry planes to construct the symmetry classes of the finite figures. The transformations of translation, screw rotation, glide reflection and rotor reflection were introduced to construct the symmetry classes of infinite figures. Infinite small translation and rotations were added to construct limiting symmetry classes. The quest for finding a single principle for the construction of any symmetrical figure was initially resolved by Wulff in 1897 and Viola in 1904 in their proof that all symmetry transformations of finite figures in three-dimensional space are reduced to successive reflections in no more than three planes, which might not be symmetry planes at all. This single principle was finally established by Boldyrev in 1907 in his proof that all symmetry transformations of finite and infinite figures in three-dimensional space are reduced to successive reflections in no more than four planes which may
not be symmetry planes of the figure. Error! Reference source not found. shows some architectural and urban design projects whose configurations are described by specific symmetry structures. These projects include Schindler’s Lowes House with one reflectional axis, Sir Sloane’s Sepulchral Church with three reflectional axes, and Kahn’s Hurva Synagogue for with four reflectional axes, Meier’s Karlsbad Apts, with a translational structure and Goff’s Price Studio with nested rotational and reflectional axes of different order. A complete presentation of the formal language to discuss symmetry will be given in the next chapter.

Figure 2-42: Configuration and symmetry. A) Schindler; b) Sir Sloane; c) Kahn; d) Meier; e) Goff

The study of configuration and the ways it informs architectural composition is an important aspect of formal analysis and synthesis in architecture composition. One of the biggest challenges that this study faces is its ability to participate actively in a variety of design contexts and not only when the presence of arithmetical or symmetrical configurations is the prevalent design solution. There is no doubt for example that any design in the Beaux-Arts tradition has to exhibit specific orders of symmetry and proportion but the challenge is to show how a better understanding of this body of knowledge – both in terms of the configurational possibilities as well as the mathematical language that describe them – can be used to any design context, even those that do not necessarily evoke such as a formal or programmatic constraint. To this turn we will turn next after we present in the next chapter the fundamentals of group theory, the mathematical language of configuration and pattern.
2.7. Summary

An overview of formal systems in architectural design has been given and their role in the systematic description, interpretation, and evaluation of existing works of architecture as well the systematic creation of new works of architecture has been described in depth. The overview here used a basic algorithmic structure for the foundation of formal systems (Stiny and Gips 1978) and all various examples of systems were presented within this framework. Two types of formal systems were reviewed, the analysis system and the design system and they were both presented as frameworks that utilize constructive and evocative modes of interpretation. The chapter concluded with an informal presentation of applications in formal analysis and design based on group theory as it pertains to the study of proportion and symmetry in architecture.

References

Fechner, G. (1876). "Elementary Aesthetics."
Chapter 2  Formal systems in architectural design..............................................6

2.1.  Introduction........................................................................................................ 6

2.2.  Formal systems .................................................................................................. 8

2.3.  The structure of formal systems....................................................................... 10

2.3.1.  Aesthetics machine....................................................................................... 10

2.3.2.  Design machine............................................................................................ 12

2.3.3.  Vitruvian machine......................................................................................... 13

2.4.  Analysis and design systems............................................................................ 18

2.4.1.  Analysis systems.......................................................................................... 19

2.4.2.  Design systems............................................................................................. 22

2.5.  Constructive and evocative systems ................................................................. 24
2.5.1. Constructive systems .......................................................... 25
2.5.2. Evocative systems .............................................................. 30

2.6. Languages and configurations ........................................... 37
   2.6.1. Proportion ................................................................. 38
   2.6.2. Symmetry ................................................................. 40

2.7. Summary .............................................................................. 44