Chapter 4  Abstraction, projection, weighting

Chapter 4 provides the hypothesis and methodology of the dissertation. Currently all formal analyses using group theoretical tools focus on repetitive designs that show immediately their symmetrical structure. It is suggested here that highly complex designs can still be described and analyzed in a group theoretical manner. The key idea is that the complexity of these designs can be seen as an aggregation of spatial layers that can all be decomposed by subgroup relations found within the symmetry of the underlying configuration.

4.1. Introduction

‘In architectural projection space is nothing more than pictures of light... Plan, section, and elevation, considered independently, are almost prehistoric. They can exist, and even coexist, without invoking projection (or indeed light) at all. A plan need not be regarded as a picture; it can just as well be thought of as a set of geometric operations on a flat sheet ’. Evans(1995)

A fascinating aspect of certain classes of architectural works is their ability to escape easy interpretations based upon existing formal tools. This is especially true for several architecture works of the modern movement that feature asymmetrical arrangements and diverse kinds of complexity. It is claimed here that a new look at existing formal tools can perform the task, if applied, though differently. The key idea here is that spatial representations of complex objects can be understood as layered compositions of simpler parts that can pictorially illustrate the symmetry structure of such a spatial configuration. It will be argued that the maximum number of layers that can be found in any spatial configuration is equal to the number of the symmetry subgroups found in the symmetry group chosen to describe the given configuration. The key idea suggested here is that the use of partial order lattice as a specific construct of formal analysis for the description of designs, along with finer distinctions of representation to account for patterns of ambiguity and emergence in the description of space, including weighting for depth and projection for transparency, can capture several properties of the description of designs that would otherwise be unnoticed. Even stronger, it is claimed here that the specific formal construct proposed here can address otherwise informal and intuitive properties of formal composition.
typically subsumed under the headings of balance, rhythm and proportion in a very specific and formal way.

4.2. Representation

‘Representation has to do with the way the objects in a computation are described...

Representation is usually divided into the verbal kind and the visual kind. The verbal kind is logical and scientific. By contrast, visual representation is characterized by the lack of primitives and by a corresponding vagueness in presentation.’ Knight and Stiny (2001)

The key ideas about the representational model are developed here. It is suggested that four aspects of representation are paramount for the formal analysis proposed here. These aspects are: a) abstraction; b) projection; c) weighting; and d) layering. Furthermore, it is suggested that these representations can be combined with one another in a group theoretical manner to accurately describe specific properties and characteristics of the formal structure of an architectural work.

4.2.1. Abstraction

‘What in art is called ‘abstract form’ ... is actually concretized conception.’ Babichev quoted in Senkevitch (1983)

In architectural design, a line drawing provides an abstracted representation of an imaginary or a real architectural object. This visual representation does not depend solely on formal similarities. This representation is not a mapping of an object’s complete form but a mapping of certain privileged or relevant aspects. Depending on the number of features being deleted from the original or alternatively being added or transformed in the mapping a level of abstraction is then arbitrarily defined. Still this is a contested territory. A typical mapping between an object and its representation is often understood as a distinction between concrete and abstract. Arnheim (1969) reworks these definitions regarding the dichotomy between concrete and abstract: ‘The abstract objects of thought, such as numbers, law, or perfectly straight lines, are real parts of nature even though they exist not as particulars...’. In gestalt theory perception of shape starts in a grasping of
generic structural features (Arnheim 1969). Perceptual organization enlists invisible extensions as genuine parts of the visible. In this context human perception is envisioned as a unitary process which leads without break from the elementary acquisition of sensory information to the most generic theoretical ideas. The essential trait of this unitary cognitive process is that at every level, it involves abstraction.

Here abstraction is used entirely in terms of essential and accidental properties (Descartes 1646). An abstracted version in any mapping is the one that keeps certain characteristics of the original object while dropping others. All such different levels of representation allow the analysis to describe shapes at different generative stages and establish links which are not immediately available to the viewer (Flemming 1990). Several types of abstraction representations are shown in Figure 4-1.

![Figure 4-1: Different levels of abstraction – a) architectonic, b) spatial, c) diagrammatic](image)

4.2.2. Projection

‘Projection is thinking of something as having properties it does not have, but that we can imagine without being conscious that this is what we are doing. It is thus a species of thought – thought about something’. Putnam (1987)

A fascinating aspect of architectural representation is its ability to describe architectural space through the deployment of depthless drawings. This paradigm shift occurred primarily in the first decades of the sixteenth century and it is exemplified by several cases where architectural projections including plans, sections, interior elevations, exterior elevations and so forth are all joining in their corresponding parts by parallel lines. These lines are ‘the agency through which the space outside the surface of the drawing is brought into it’ (Evans 1995). Thus architectural projection, becomes ‘nothing else other than the finest light’ (Panofsky 1968); the images drawn
as if transmitted to a surface by light appear flattened into a comb of drafted lines. Since then, light has become the ultimate geometric instrument, and lines are identified to light paths (Evans 1995). The geometry of images propagated by light has since been developed as a postscript of that of land survey.

More importantly, the unintended consequence of all these linear connections of representations, and in essence parallel computations, was to create a new kind of representation that grasped the imaginary space behind the original drawings and opened up to space. This showdown, obscured by an architectural space limited to the pictures that gave access to it, finally opened up to the parallel, orthographic projection of the architecture that brings space into pictures. Since then, a plan ceased to be regarded as a picture and came into being as a set of geometric transformations on a flat sheet.

Significantly, these initial computations with shapes were not about descriptions of forms of buildings as extant constructions of physical materials in physical space. Rather, they were descriptions of designs to become ‘constructions of imagination’ (Mitchell 1990). In this way, these initial computations opened up the way to think nowadays of the space populated by collections of graphic tokens such as points, lines, and polygons as two-dimensional and three-dimensional design worlds. And among all these worlds, the $U_{12}$ world, the design world consisting of lines in the plane, is assuming an authority that maps directly from Alberti’s world directly to the world of CAD systems.

Still these lines – and all lines in any spatial system – come with a baggage; they stand for orthographic and perspective projections and the choice of the system is often ambiguous. Here the main concern deals with a specific kind of combined orthographic projections through descriptive geometry and in particular the specific composite drawing technique consisting of orthographic and affine transformations. This representation is quite unique in architecture and not in other engineering fields, for of its ability to represent abstract space both visually and metrically. This type of composite projection consisting of orthographic and affine transformations produces what is nowadays understood as ‘paraline drawings’ both axonometric and oblique (Uddin 1997).

Paraline drawings are capable of creating angular variety and having different emphasis, using the same plan and elevation-section. In an oblique projection, the surface parallel to the picture plane remains to its true size while the other two visible surfaces of the object
become foreshortened. Paraline drawings offer a suitable alternative to perspective and orthographic projection because they combine plan, elevation and section in one three-dimensional drawing where lines are drawn to a scale. Axonometric projection refers to the image being foreshortened as a result of inclined and rotated portion of the object with respect to the picture plane. In contrast, the desired appearance of an oblique projection depends on three factors: orientation of the primary orthographic projection, angle and direction of receding lines, and ratios of foreshortening for vertical planes. This type of parallel projection is efficient for complex architectural representation, due to its versality, flexibility, and capability to show more information and support designing more than any other medium. Several types of axonometric and composite axonometric views are shown in Figure 4-2.

![Figure 4-2: Types of axonometric projections – a) initial state; b) plan oblique; c) elevation oblique](image)

The nature of the images generated by paraline drawings attracted the interest of twentieth century architects because seeing in oblique views is essential for depth perception. It is significant that the neo-plasticians and the suprematists manufactured the comeback of parallel projection into the central stage of modern architecture. It is also with no surprise that the revival of the 1920s in the 1960s relied upon architectural axonometric as the primary medium of representation and it was carried to new heights by the New York Five (Deamer 2001) and their interpretations of the organizations of classical vocabularies of architecture including aspects of rectangularity, planarity, axiality, symmetry, and frontality (Evans 1995). It is significant as well that these latter appropriations of composite axonometric projections suggested two functions initially thought as one: first, the axonometric as a presentational device, and second as a conceptual device. The former is progressively enriched by the methods of multi-media presentation, the latter is able to support the communication of spatial qualities of an architectural object during the process of conceptualization. In all cases the exploitation of the fundamental ambiguity of the axonometric projection has been the discourse of much architecture in the twentieth century and it is suggested here that this should be the primary mode of representation to adequately describe and analyze the architecture works of these corresponding periods.
4.2.3. Weighting

*The physical quality involves the visual effect of an appearance of weight and mass, or the force of gravity, acting on a form.* Ladovsky (1926)

Weights are familiar in architecture. A pen stroke has a width. Lines are of different thickness. Typically these weights refer to what is represented and work upon a set of conventions widely shared and understood. Different types of conventions and assumptions often give the clues about what is represented. For example, Dokuchaev suggested that three types of formal manipulation can provide perceptual clues about the apparent mass and weight of a form: surface treatment, surface details, and formal allusion (Senkevitch 1983). In the first type, the surface treatment consists of the use of smooth surfaces to convey a feeling of lightness and roughly textured surfaces to make a form appear more massive. The second type involves the use of surface details, ranging from the fluting on a column to the joints in a wall and layering and interpenetration of planes in the surface of an abstract form. The third type of manipulation, formal allusion, is directly toward affecting the appearance of the form as a whole. Such manipulation ranges from the use of a single form, such as an inverted cone, to suggest the movement downward of a single force, to the conscious deformation of a simple or compound form.

Klee (Spiller 1961) and Stiny (1992), albeit in different ways, set the emphasis in the representation itself rather than what is represented. In this discourse, weights distinguish formal attributes from physical properties. Shapes made up of basic elements and weights answer to the Vitruvian categories: physical properties being included in firmness, functional properties in commodity, and spatial properties in delight. Several types of weighted representations are shown in Figure 4-3a below.

The lines, as apparatus, enable other conceptual functions. In architecture, the line is the means by which architecture displays its conceptual accretions. The intersection of the linear economy with a built form is Alberti’s remarks on the architectural wall (Alberti 1955). The wall is represented in the architectural section as a double line, as a path. Note that the edge is simply represented by a single line. Sectioned solid walls are mapped to solid | thick lines; in view, standard | thin lines outline contours; projected, contours are represented by single line, while close edges (beam) are represented by double dotted lines. Beneath this presentational level, more investigation is needed in the conceptual level. Meier’s drawings – plans, sketches, and working
drawings – are scalar analogs that serve to visualize or predict the experience of architecture, and are highly abstract notational scores that condense amounts of information – both visual and non-visual – into a codified language of symbols. A line on a plan may mark the separation of inside and outside, but it can also signify the edge of a volume, a change in material or level, or the presence of something above or beyond. Or it may indicate something not physical at all: axis of view, an alignment in space, or trajectory of movement. Solid, void, or glazing can all be noted with a limited catalog of linear marks. In part the clues to read these marks are contextual, fixed by the local syntax, and in part they are conventional, based on architecture’s accumulated memory.

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Figure 4-3: Types of weighted projections – a) Line weight notation; b) Axonometric line weight

This particular understanding of weights as a characteristic of representation itself is taken here as a powerful construct for formal analysis. In this latter view and according to Klee, weights suggest a different kind of projection, whereas ‘the irregular projection consists in the accentuation of parts or the omissions of certain parts’ (Spiller 1961). This type of representation directly alludes to transparency, a key feature of the twentieth century architecture, whereas the three-dimensional qualities of the design are illustrated through the weighting of lines. Varying degrees of transparency and opacity can be strategically deployed to hierarchically differentiate
preselected conditions important to the development of the work. The axonometric projection provides an opening in the hands of say, Lissitsky (1968) in his Prouns or Van Doesburg in his Counter-Compositions (Padovan 2002), that enables an access to a new kind of space proper to the new era by exploiting the ambiguity of spatial registration. The representation suggested here uses line weights as an integral part of the underlying mechanism of representation to adequately describe and foreground transparency in architectural works.

4.2.4. Layering

'Space, not stone, is the material of architecture. It is in space that the soaring wonders of modernity will be built by art plus the intellect.' Ladovsky (1920)

Spatial form is often conceived as a composite of layered planes making up its envelope and giving form to characteristics. The envelope of an architectural form possesses actual depth, like the wall of a building, and bears a certain correspondence with the idea of building up form in parallel vertical layers and to the dynamic planar constructions. Several examples would do but among the different architectural discourses the constructivism in particular abounds with exemplary case studies - see for example, Tatlin’s counter-reliefs (Senkevitch 1983). This aggregation entails building up the envelope out of a network of layered planes, overlapping and intersecting at various coordinated angles. The envelope would thus acquire not only an essential three-dimensional aspect, but also a plastic quality, obtained from the interaction of receding and advancing planes and the rhythmic flow of their lines. All planes derive their ultimate impact and significance from being combined and arranged in equilibrium to create an integral architectural whole. The articulation of surface and form as envelope reveals how crucial the geometrical articulation of form was to the attainment of a lucid equilibrium: the shapes, sizes, and proportions of the constituent planes and intervals all had to be balanced and correlated with absolute precision.

This interpretation of a design as a system of boundary elements leads to a nested hierarchy of representations that start from definitions of bounded envelopes, continue to permeable envelopes and end to architectonic envelopes. The wall in all its instantiations is a prime determinant of spatial form and provides a major key of the solution to comprehend complexity. Fundamental problems here are found in both passages from one level of representation to the next. For example, the articulation of the permeable envelope as a prime
The determinant of spatial form is inextricably linked to the prior definition of the boundary elements. The concept of permeability or alternatively the degree of a form’s openness to circulation plays a major role in composition. The rationalists saw a dynamic level of permeability and the interpenetration of space that it suggests as a vital means for emphasizing the esthetic and perceptual power of space. Appropriately, the articulation of an architectonics envelope as an equally significant prime determinant of spatial form is inextricably linked to the prior definition of the permeable elements.

Here two interpretative ways of the wall function as a distributor of the forms of architecture are briefly discussed: the ‘articulation in relief’ (Ladosky 1926) and the ‘organizational scaffolding’ (Deamer 2001). The articulation in relief is the constructivist device to enclose the volume of the wall as a composite of layered planes. This technique has been proposed for constructing the envelope as an aggregate of planes, and is what Ladovsky calls articulation in relief. That kind of envelope possesses actual depth and bears Hildebrand’s (1893) idea of building form in parallel vertical layers. This idea stresses the aesthetic of perception where the essence of architectural solution leads to ‘the controlled modulation of spatial magnitudes’ through means of expression such as mass, weight, color, proportion, movement, and rhythm. Another idea coming from Tatlin’s (1915) counter-reliefs seeks to integrate into a spatial whole space within the construction and space beyond. Krinsky’s (1921) Tribune Project materializes that approach.

On the other hand, the organizational scaffolding can be seen as the primary conceptual device of late-modern architects (Deamer 2001). It emphasizes the surface as an updated version of Gestalt theory articulated by Slutzky’s painterly two-dimensional surface and in particular his work on phenomenal transparency depending on two-dimensional surface (Deamer 2001). Equally important is the work of their successors: it is the surface Libeskind “writes” on, it is the surface the computer turns into folds, it is the surface that advanced digital techniques turn to digital landscapes and topographies (Deamer 2001). Analogously, for the New York Five architects, this scaffolding to distribute the forms of architecture is the implied grid that locates the datum, the regulating lines, the location of frontal or rotated plane or surface, and the layering (Deamer 2001). For the current computer-based architecture, the scaffolding is still an a priori plane or surface that pre-determines the distribution of forms, thereby sparing the architect the need to make any arbitrary move.
Here a synthesis of these layering strategies is attempted to account for a new layering strategy that borrows elements from both and organizes the whole construct along the premises of a group theoretical approach. In this case, the articulation in relief and the organizational scaffolding are developed around the partial order lattice of the configuration of the overall design and carry through the computation at any level desired. An example of layering mapping is shown in Figure 4-4. In this example, a decomposition of a design in terms of some parts in an initial representation parses the design in constituent parts that are all keeping their independence in any other representation.

Figure 4-4: Two types of schematic drawings showing layered mappings. – a) Tatlin and Krinsky’s counter-reliefs: sculpture and facade ; b) Jeanneret and Meier’s scaffoldings: painting and façade
4.3. Partial order

‘The semi-lattice is the structure of the complex fabric; it is the structure of living things – of great paintings and symphonies.’ Alexander (1965)

All interpretations of designs suggest a decomposition of perceivable units and fundamental elements used or observed in their construction. Often these elements and units are identified with boundary elements and their arrangements, in particular walls and their assemblies. Here the key operator that parses the representations discussed above is the partial order relation defined by the symmetry group that describes the maximum symmetry of the configuration. The fundamental significance of symmetry arises here from its capacity to reveal two opposing aspects of form: transformation (change) and conservation (invariance). That which is conserved during a change is an invariant; the set of transformations which keeps something invariant is its symmetry group. The set of elements and their structural relationships forming the complete system are conserved as a single whole and this order identifies all the nested parts in any configuration that have a group theoretical relationship to the overall group of the configuration.

The key idea is that spatial representations of complex objects can be understood as layered compositions of simpler parts and these parts can all be related through symmetry values from group theory. These values can be structured as a partial order lattice that pictorially presents the symmetry structure of any spatial configuration; the number and qualities of the symmetry subgroups found in any given configuration provide the maximum number of layers that can be found in a spatial configuration; for example, in any spatial arrangement that is based on the structure of the square the maximum number of layers and spatial constructs that can be build upon those is ten because this is the number of symmetry subgroups of the square. Figure 4-5 shows the partial order of the square.

Figure 4-5: A partial order lattice of the square: a) set notation; b) discursive notation
4.4. Model


The key hypothesis is that spatial representations of complex objects can be understood as layered compositions of simpler parts and these parts can all be related through symmetry values from group theory. These values can be structured as a partial order lattice that pictorially presents the symmetry structure of any spatial configuration; the number and qualities of the symmetry subgroups found in any given configuration provide the maximum number of layers that can be found in a spatial configuration; for example, in any spatial arrangement that is based on the structure of the square the maximum number of layers and spatial constructs that can be build upon those is eight because this is the number of symmetry subgroups of the square. Still, the symmetry subgroups can only provide the logical framework to compute an architectural composition; what is critical is the representation of the designs that are going to be analyzed within this framework. The model suggests four parts: a) a stop mode (□); b) a rewind mode (<<); c) a play mode (>); and d) a forward mode (>>) and all four are discussed below.

4.4.1. Stop mode

The stop mode freezes the design in one representation that will provide the blueprint for the formal analysis. This is often a contested ground as design is rightly considered as an element that emerges among many different kinds of descriptions and there is no ultimate description of a design (Stiny, 1992). Here this initial design is considered to be the mapping of a collection of descriptions in a three-dimensional geometrical model that complies with the given drawings, in the fullest possible degree. The rendering of the model is then taken in a transparent axonometric view as a medium of architectural projection.

4.4.2. Rewind mode

The rewind mode entails an arbitrary number of successively abstracted models based on the initial three-dimensional model of the stop mode. In principle there can be an infinite number of abstracted models that are successively removed from the original one. In theory there is a need for only one more additional model to show the transfer of the parsing of the one model to the
other. Still here, three levels of notational languages are suggested as a minimum for formal analysis. The first level of notation, the architectonic level, is the level that approximates in some way the original notation used for the language of the actual design model into its closest geometric representation. The notation privileges functional elements such as walls, slabs, columns, and beams, walled furniture, handrails, and openings of various kinds such as windows, doors, stairwells, chimneys, and so forth. All elements are weighted and usually represented as architectural conventions indicate. A weighted model contains the whole set of architectural features, i.e. a collection of information with values made explicit by the designer. Wall tectonics, physical properties, spatial functions and all kind of features to be expected in design models. This level concretizes the geometric features into their material expression (Alberti 1955). The units of the surface structure belong to a repertoire that constitutes the visual vocabulary of the designer.

Figure 4-6: Example of architectonic notation

The second level of notation, the spatial level, privileges space divisions and corresponding openings in these boundaries, and discards all other information. This level essentially picks up planes that function as walls and slabs and so on, the connections between them and their interface with context for ventilation, light and so forth. At this level, a spatial model that emerges is a spatial decomposition of the building with geometric shapes that bound the space. By inspection, a greater simplification of the walls should help to determine the small set of basic instances derived from the generic wall that constitute the constructive vocabulary of designer’s art of generating space.

Figure 4-7: Example of spatial notation
The third level of notation, the diagrammatic level, foregrounds underlying, emergent boundaries of space and discards all connections between them. This level of notation, closely related to the parti of a design, the geometrical diagram or pattern that emerges when all details have been dropped out, is the most abstract version of the model and functions as a scaffolding of the design. Metric distances between boundaries are taken into account. Vitruvius defines ordering as the initial commitment to a geometrical system that controls the subsequent design, usually a modular layout (not necessarily a grid), because it consists of deciding the quantity of the module, and unites the individual parts to the overall proportional system.

4.4.3. Play mode

The play mode is the application of group theoretical analysis in the simplest representation that has been found. All parts of the design that comply with the symmetry group of the configuration and all the symmetry subgroups are extracted and ordered in partial order lattices. The layered arrangement of all the parts the one upon the other should be able to give the complete simplest configuration that started the computation.
4.4.4. Fast Forward mode

The forward mode entails the reversal of the process from the simplest representation to the initial three-dimensional model. In this latter case the parts of the design that were parsed in the previous stage still come parsed in the successive layers and suggest correspondences that otherwise would be impossible to observe.

Figure 4-10: A partial order lattice of a complex arrangement

4.5. Conclusion

The challenging urgency was to define critical constructs of the late-modern designs in order to lay out the proper process of the group theoretical formalism. The best way now to test the validity of this methodology is to justify a selection of some representative of the late-moderns and to algorithmically analyze it. What can be implemented on the Smith house of Richard Meier.

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